

Preparing for the final:

You will be allowed to use ONE 8 ½ by 11 study guide on the final! You may only use ONE side (not both). You can hand write or type your study guide. Study guide MUST be approved by your geometry teacher BEFORE you take the final. If you fail to follow these directions you will NOT be allowed to use your study guide on the final! When in doubt, ask!

What you can include:

- Formulas/Rules
- Definitions
- Ideas/concepts
- Diagrams

Not allowed:

- Unit circle
- Worked out examples

Here are two examples:

Example 1:	<p><i>The volume of a cube is 64 inches cubed. What is the area of one face of the cube?</i></p> $V = s^3$ $64 = s^3$ $s = 4$ $A = 4 * 4 = 16$	Allowed? Why or why not?
Example 2:	<p>$V(\text{cube}) = s^3$</p> <p><i>If given volume of a cube, remember to take cube root to find length of ONE side</i></p>	Allowed? Why or why not?

Here are the units we have studied 2nd semester. The review packet will follow in this order, as well as the final. Whatever helpful hints you think you might need, include on your study guide!

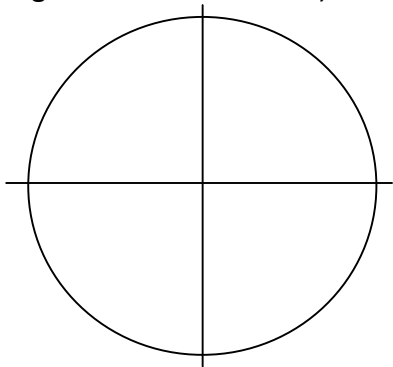
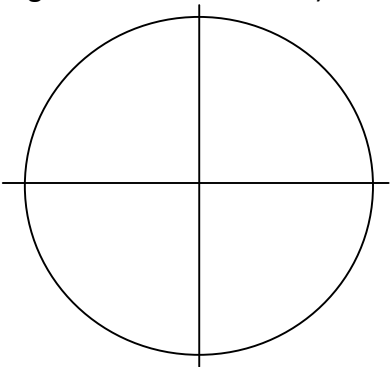
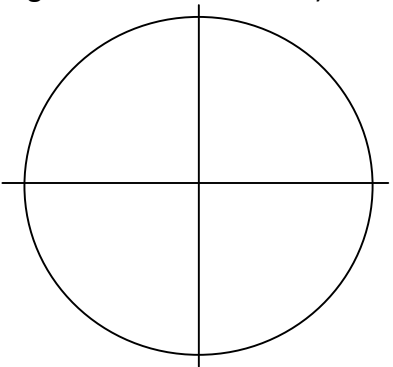
- 1) **Unit Circle***
- 2) Properties of Circles
- 3) Perimeter/Area
- 4) Surface Area/Volume
- 5) Properties of Quadrilaterals
- 6) Transformations
- 7) **Rational Expressions***

*These units will be particularly important to review as they will be apart of the Pre-Calc curriculum next year.

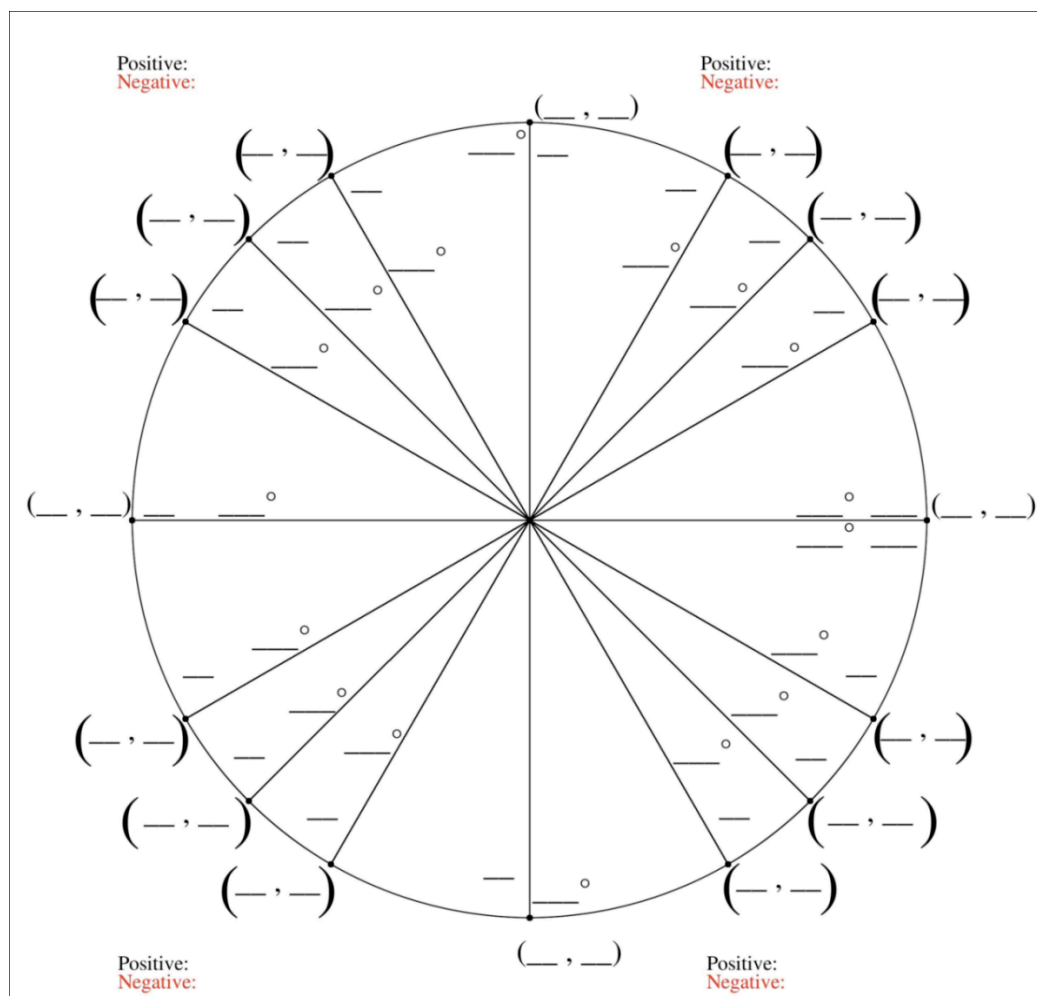
Good Luck!

1) Review Unit Circle

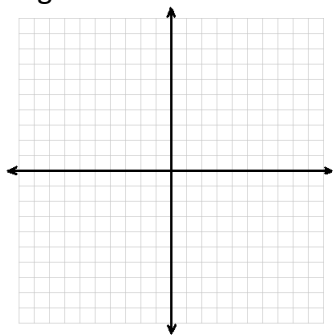
Use right triangles to derive the coordinate points in the first quadrant of the unit circle.

<p>1) $\theta = 45^\circ$ (draw a special right triangle for each unit circle)</p>  <p>A unit circle with a vertical line from the center to the top and a horizontal line from the center to the right, forming a cross. The circle is divided into four quadrants.</p> <p>$\cos 45^\circ =$</p> <p>$\sin 45^\circ =$</p> <p>Coordinate =</p>	<p>2) $\theta = 30^\circ$ (draw a special right triangle for each unit circle)</p>  <p>A unit circle with a vertical line from the center to the top and a horizontal line from the center to the right, forming a cross. The circle is divided into four quadrants.</p> <p>$\cos 30^\circ =$</p> <p>$\sin 30^\circ =$</p> <p>Coordinate =</p>	<p>3) $\theta = 60^\circ$ (draw a special right triangle for each unit circle)</p>  <p>A unit circle with a vertical line from the center to the top and a horizontal line from the center to the right, forming a cross. The circle is divided into four quadrants.</p> <p>$\cos 60^\circ =$</p> <p>$\sin 60^\circ =$</p> <p>Coordinate =</p>
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Fill in the unit circle:



4) Draw a 50° angle.

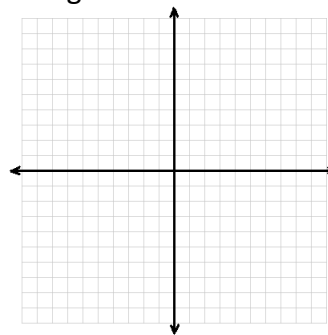


a. Find **two** positive angles that are coterminal with the given angle.

b. Find **two** negative angles that are coterminal with the given angle.

c. Convert 50° to radians.

5) Draw a -230° angle.



a. Find **two** positive angles that are coterminal with the given angle.

b. Find **two** negative angles that are coterminal with the given angle.

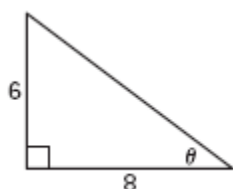
c. Convert -230° to radians.

6) Convert $\frac{3\pi}{2}$ radians to degrees.

7) Convert $\frac{4\pi}{3}$ radians to degrees.

NAME	ABBREVIATION	RATIO	NAME	ABBREVIATION	RATIO
sine			cosecant		
cosine			secant		
tangent			cotangent		

8) Evaluate the six trigonometric functions of the angle θ .



9) Let θ be an acute angle of a right triangle. Find the value of the other five trigonometric functions of θ .

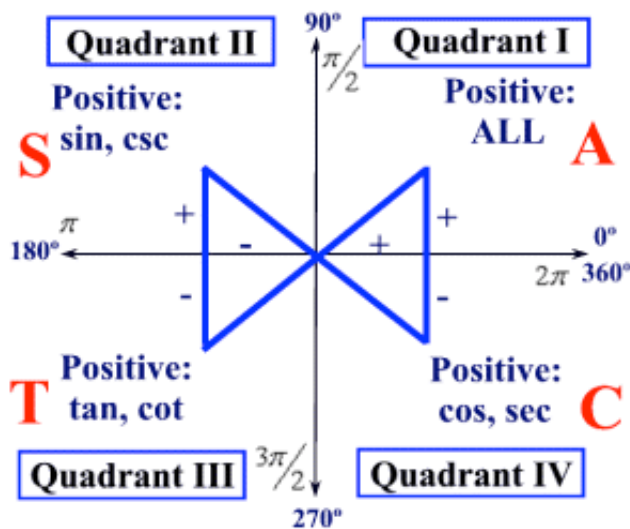
$$\sin \theta = \frac{\sqrt{2}}{2}$$

10) Evaluate the 6 trig functions of 45°

11) Evaluate the 6 trig functions of 30° .

If θ is not a quadrantal angle, the sign of a trig function depends on the quadrant in which θ lies:

Draw Picture:



“A Smart Trig Class” or “All Students Take Calculus”
A S T C

*All positive in Q1

*Sine (and reciprocal function cosecant) positive in Q2

*Tangent (and reciprocal cotangent) positive in Q3

*Cosine (and reciprocal secant) positive in Q4

12) Evaluate, if possible, the sine function and tangent function of the 4 quadrantal angles:

a. $\theta = 0^\circ = 0$

b. $\theta = 90^\circ = \frac{\pi}{2}$

c. $\theta = 180^\circ = \pi$

d. $\theta = 270^\circ = \frac{3\pi}{2}$

13) Name the quadrant in which angle θ lies.

a. $\sin \theta < 0$ and $\cos \theta < 0$

b. $\tan \theta < 0$ and $\cos \theta < 0$

c. $\tan \theta < 0$ and $\sin \theta < 0$

d. $\cot \theta > 0$ and $\sec \theta < 0$

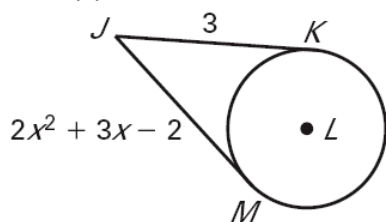
<p>14) Find the reference angle θ', for each of the following angles:</p> <p>a. $\theta = 210^\circ$</p> <p>b. $\theta = \frac{7\pi}{4}$</p> <p>c. $\theta = -240^\circ$</p>	<p>15) Use reference angles to find the exact value of the following trig functions:</p> <p>a. $\sin \theta = 300^\circ$</p> <p>b. $\tan \theta = \frac{5\pi}{4}$</p> <p>c. $\sec \theta = (-\frac{\pi}{6})$</p>
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Evaluate the function without using a calculator.

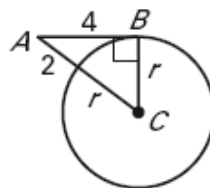
16) $\sin 240^\circ$	17) $\cos 330^\circ$
18) $\sin \frac{4\pi}{3}$	19) $\cos \frac{7\pi}{6}$
<p>20) Solve the equation for θ without a calculator. Give your answer in both radians and degrees.</p> $\sin \theta = (-\frac{\sqrt{3}}{2})$	<p>21) Solve the equation for θ without a calculator. Give your answer in both radians and degrees.</p> $\tan \theta = (-\sqrt{3})$
<p>22) Solve the equation for θ without a calculator. Give your answer in both radians and degrees.</p> $\cos \theta = (-\frac{1}{2})$	<p>23) Solve the equation for θ without a calculator. Give your answer in both radians and degrees.</p> $\sin \theta = 0$

2) Review Circles

1) The points K and M are points of tangency. Find the value(s) of x .

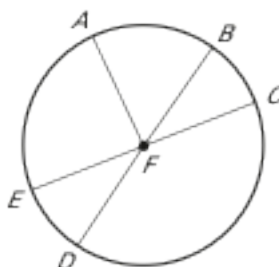


2) Find the radius, r .



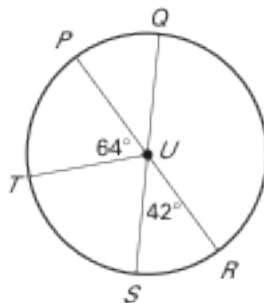
In $\odot F$, determine whether the given arc is a *minor arc*, *major arc*, or *semicircle*.

1. \widehat{AB}
2. \widehat{AE}
3. \widehat{EAC}
4. \widehat{ACD}
5. \widehat{CAD}
6. \widehat{DEB}
7. \widehat{BAE}
8. \widehat{DEC}



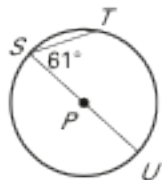
In the figure, \overline{PR} and \overline{QS} are diameters of $\odot U$. Find the measure of the indicated arc.

9. $m\widehat{PQ}$
10. $m\widehat{ST}$
11. $m\widehat{TPS}$
12. $m\widehat{RT}$
13. $m\widehat{RQS}$
14. $m\widehat{QR}$
15. $m\widehat{PQS}$
16. $m\widehat{TOR}$
17. $m\widehat{PS}$
18. $m\widehat{PTR}$

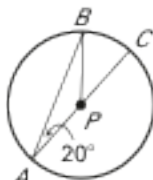


Find the measure of the indicated angle or arc in $\odot P$.

2. $m\widehat{ST}$



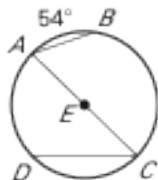
3. $m\widehat{AB}$



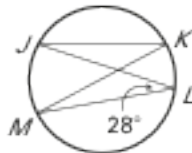
4. $m\angle JLM$



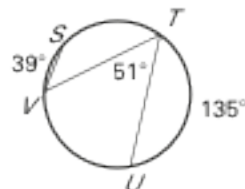
5. $m\angle A$



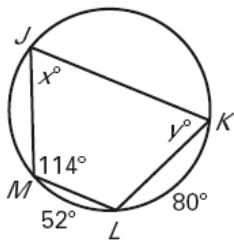
6. $m\angle K$



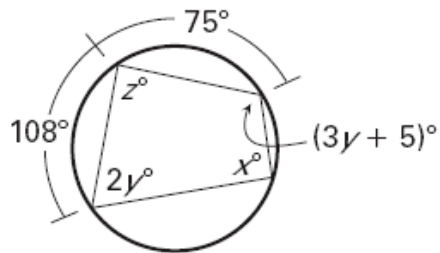
7. $m\widehat{VST}$



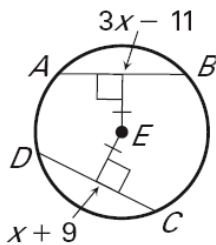
8) Find the values of the variables.



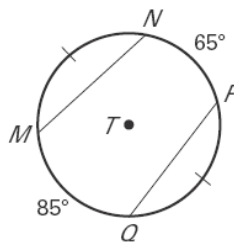
9) Find the values of the variables.



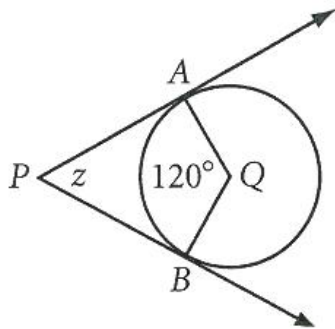
10) Find the value of x in the figure below.



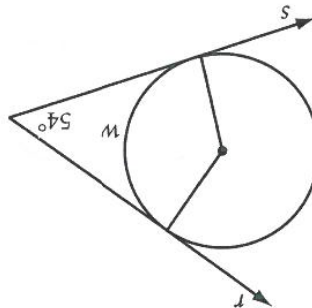
11) Find the measure of arc MN.



12) PA and PB are tangent to circle Q. Find z .



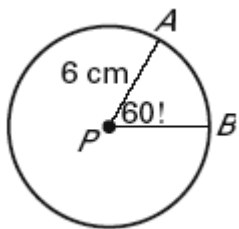
13) Lines r and s are tangent to the circle. Find w .



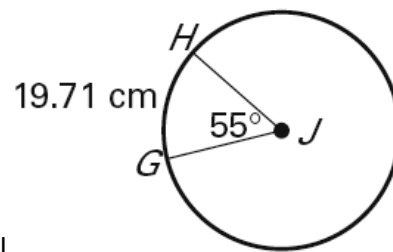
14) A pizza is sitting in a circular box that fits its edges exactly. The pizza has a diameter that is 12 inches across. The pizza has 8 equal slices cut at equal angles through the center of the pizza. When you take one piece of pizza from the circular pizza box, what is the approximate length of the exposed side of the box?

15) A child's bicycle wheel completes exactly three revolutions when it travels 150 cm. The wheel has a decoration on it that covers exactly half of the inner part of the wheel. What is the area of the decoration? Leave your answer in terms of π .

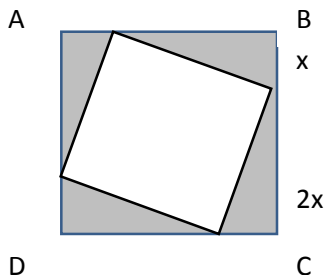
16) Find the length of arc AB.



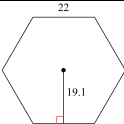
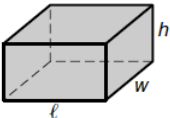
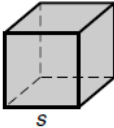
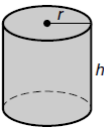
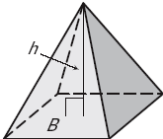
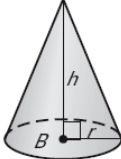

1) Find the circumference of circle J.



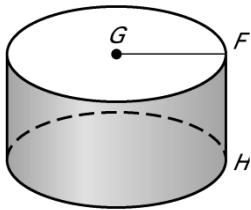
3) Review Perimeter/Area

<p>1) A farmer who owned a 20-yard-by-40-yard plot of land purchased more property such that the area doubled. The 40-yard length of the plot increased by 10 yards. How much must the 20-yard width have increased?</p>	<p>2) The volume of a cube is 64 inches cubed.</p> <p>a) What is the area of one face of the cube?</p> <p>b) What is the full surface area of the cube?</p>
<p>3) A square and a semicircular region have the same perimeter. If the length of the radius of the semicircular region is 8, what is the length of one side of the square?</p>	<p>4) A thumbnail of a photographer's image shows a certain skyscraper to be 5 centimeters tall. In actuality the skyscraper is 400 meters tall. What is the scale factor of the building to the thumbnail?</p>
<p>5) The Jones family was planning on taking a road trip from Chicago to Madison, WI, which is 500 miles. Instead, they decided to go to Springfield, IL, which is 630 miles from Chicago. On a certain map, each inch equals 60 miles. On that map, how much longer is the trip to Springfield than the trip to Madison?</p>	<p>6) Brandon is going to cover his bathroom with tiles, and he plans to but the tiles next to each other so there is no space in between them. The tiles are rectangular prisms that are 2 centimeters tall by 10 centimeters wide by 8 centimeters long. If Brandon's bathroom is a square that measures 4 meters by 4 meters, what is the minimum number of tiles he will need to fully cover his bathroom floor?</p>
<p>7) EURO advertising company sells advertising space of the bottom of the inside of pizza boxes from Pisano's. If the box for the 14 inch pizza is a square and leaves room for one inch on each side of the pizza, approximately how much advertising can be seen BEFORE the pizza is eaten?</p>	<p>8) In the figure below, ABCD is a square. Points on each pair of adjacent sides of ABCD are connected to form 4 congruent right triangles with one leg two times as long as the other, as shown below. What fraction of the area of square ABCD is shaded?</p> 

4) Review Surface Area and Volume

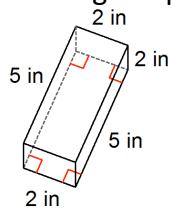
Solid	Formula: SURFACE AREA	Formula: VOLUME	Picture
REGULAR POLYGON			
RECTANGULAR PRISM			
CUBE			
CYLINDER			
PYRAMID			
CONE			
SPHERE			

13) Use the diagram at the below to answer the questions at the right.



- a. Give the mathematical name of the solid.
- b. What kind of figure is each base?
- c. Name the radius of the solid.
- d. Name the height of the solid.

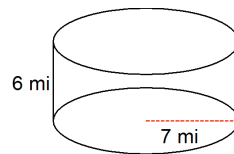
14) Find the surface area and volume of the rectangular prism below.



Surface Area:

Volume:

15) Find the surface area and volume of the cylinder below and leave it in terms of π .



Surface Area:

Volume:

16) A cube has a volume of 1728 cm^3 .

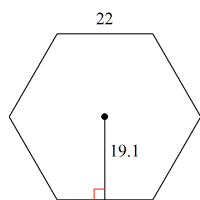
a. What is the area of one face of the cube?

b. What is the full surface area of the cube?

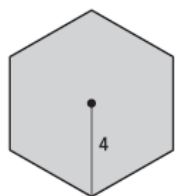
17) The surface area of one face of a cube is 121 m^2 . Find the volume of the cube.

18) Find the area of the regular polygons below. All units are in inches.

a.



b. Find the area of the regular polygon below. Each side is 5 cm. Round to the nearest hundredth.



19)

a. In #1 problem (a) I was given (circle 2):

Apothem "Radius" Side length

So I solved by _____

 _____.

b. In #1 problem (b) I was given (circle 2):

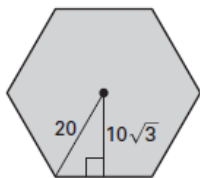
Apothem "Radius" Side length

So I solved by _____

 _____.

Continued →

c.



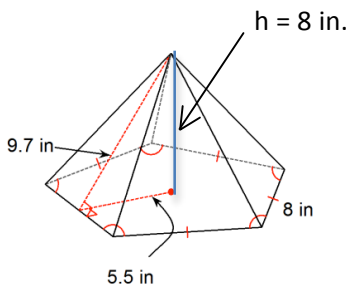
a. In #1 problem (c) I was given (circle 2):

Apothem "Radius" Side length

So I solved by _____

 _____.

20) Find the surface area and volume of the pyramid below.



Surface area: _____ Volume: _____

21)

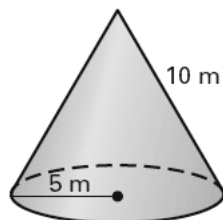
- a. Which of the values shown in # 3 is the **lateral** height of the pyramid?

- b. Which of the following formulas requires **lateral** height for a pyramid—surface area or volume?

- c. Which of the values shown in # 3 is the **actual** height of the pyramid?

- d. Which of the following formulas requires **actual** height for a pyramid—surface area or volume?

22) Find the surface area and volume of the cone below.



Surface area: _____ Volume: _____

23)

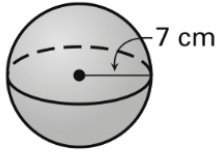
- a. Which of the values shown in # 5 is the **lateral** height of the cone?

- b. Which of the following formulas requires **lateral** height for a cone—surface area or volume?

- c. Which of the values shown in # 5 is the **actual** height of the cone?

- d. Which of the following formulas requires **actual** height for a cone—surface area or volume?

24) Find the surface area and the volume of the



Surface area: _____ Volume: _____

25) State the error based on # 7.

$$V = \frac{4}{3} \pi (7)^2$$

$$V = \frac{4}{3} \pi (49)$$

$$V = 579.1 \text{ in}^3$$

Error: _____

26) The length of each side of a cube is increased by 4 times. By how many times did the volume increase?

27) A cylinder has a radius of 4 ft and height of 10 ft. If the radius is increased by 2 times, find the ratio of the old volume to the new volume.

28) The length of each side of a cube is multiplied by 3. What is the change in *volume* of the cube?

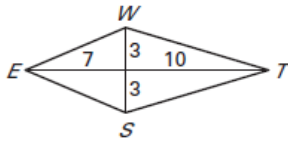
- A. The volume is 3 times greater.
- B. The volume is 6 times greater.
- C. The volume is 9 times greater.
- D. The volume is 27 times greater.

29) A cylinder has a height of 4 cm and a radius of 2 cm. If both the height and radius are doubled, what is the ratio of the volume of the original cylinder to the volume of the new cylinder?

5) Review Properties of Parallelograms

<p>1) Which of the following statements is NOT true about parallelograms?</p> <p>a. consecutive angles are congruent b. opposite sides are congruent c. opposite angles are congruent d. the diagonals bisect each other</p>	<p>2) Which of the following quadrilateral is not a parallelogram?</p> <p>I. Trapezoid II. Kite III. Square IV. Rectangle</p> <p>A. I B. II and III C. I and II D. I, II, and III E. All of above</p>
<p>3) If one diagonal of a rhombus is 10 cm and the other is 24 cm.</p> <p>a. How long is each side of the rhombus?</p> <p>b. Find the perimeter.</p> <p>c. Find the area.</p>	<p>4) The diagonal of a square is 14 inches.</p> <p>a. How long is each side of the square?</p> <p>b. Find the perimeter.</p> <p>c. Find the area.</p>

5) Use the figure below to:

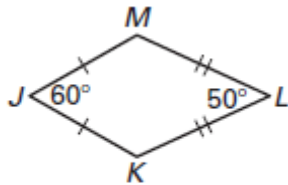


a. Find the length of each side.

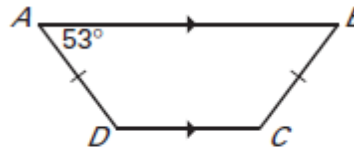
b. Find the perimeter.

c. Find the area.

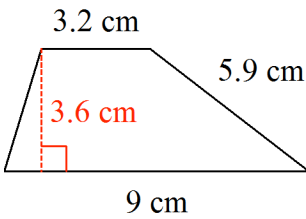
6) Find the measurement for angles M and K.



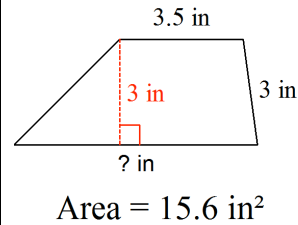
7) Find the measurements of angle B, C, and D.



8) Find the area:

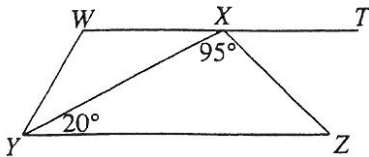


9) Find the base:

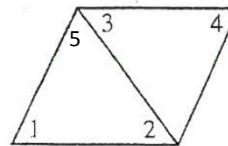


10)

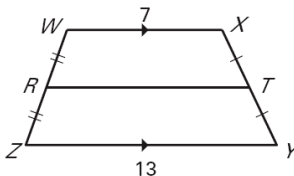
In the figure below, $WXZY$ is a trapezoid, point X lies on WT , and the angles are as marked. What is the measure of $\angle ZXT$?



11) In the parallelogram below, a diagonal is shown and $\angle 3$ measures 40° and $\angle 5$ measures 65° . What is the $m\angle 1$?



12) Find the midsegment RT .



6) Review Transformations:

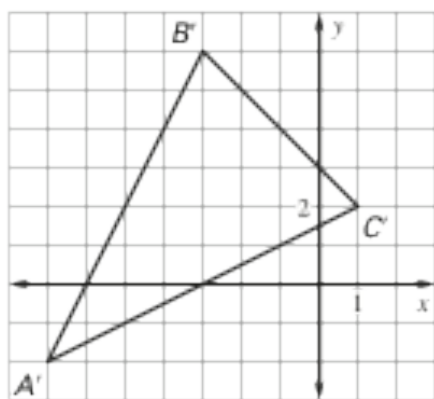
1) Multiply:

$$\begin{bmatrix} 3 & -1 \\ 5 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$$

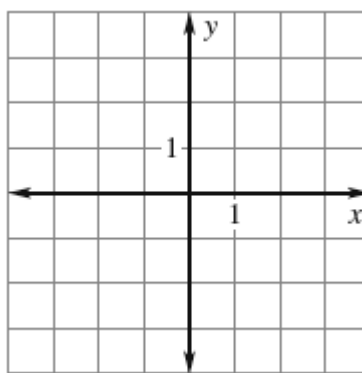
2) $\triangle XYZ$ with vertices $X(-2, 4)$, $Y(6, 2)$, and $Z(3, -2)$ is translated to $\triangle X'Y'Z'$. Determine the translation using a vector in component form, and determine the coordinates of the remaining vertices. $Z'(-2, 2)$

3) Use the described translation and the graph of the image to find the matrix that represents the preimage.

4 units right and 2 units up

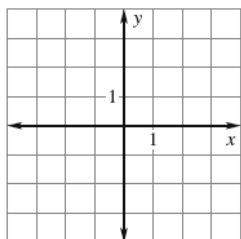


4) The vertices of $\triangle ABC$ are $A(2, -1)$, $B(-2, 2)$ and $C(1, 2)$. Reflect the segment in the line $y = x$. Graph the figure and its image.

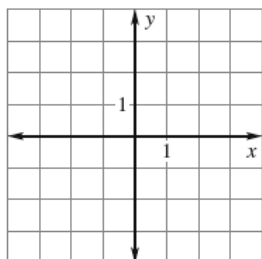


5) The endpoints of JK are $J(-1, -2)$ and $K(1, -2)$. Reflect the segment in the given line. Graph the segment and its image.

a. $y = 1$



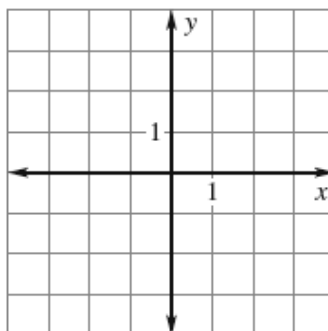
b. $y = -x$

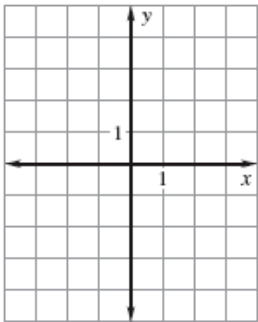
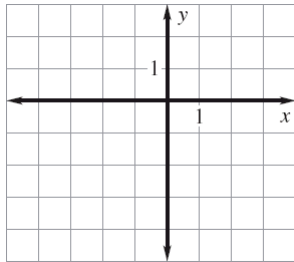
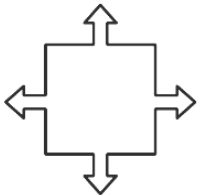
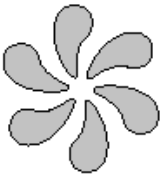


6) The vertices of $\triangle QRS$ are $Q(-1, 4)$, $R(0, 1)$, and $S(2, 3)$.

a. Find the reflection of $\triangle QRS$ in the x -axis using matrix multiplication.

b. Graph $\triangle QRS$ and its image.



<p>7) What are the coordinate point rules for rotations counterclockwise about the origin?</p> <p>a. For a rotation of 90°, $(a, b) \rightarrow (\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$.</p> <p>b. For a rotation of 180°, $(a, b) \rightarrow (\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$.</p> <p>c. For a rotation of 270°, $(a, b) \rightarrow (\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$.</p>	<p>8) What are the matrices for rotations counterclockwise about the origin?</p> <p>a. For a rotation of 90°, (a, b)</p> <p>b. For a rotation of 180°, (a, b)</p> <p>c. For a rotation of 270°, (a, b)</p>
<p>9) Trapezoid $DEFG$ has vertices $D(-1, 3)$, $E(1, 3)$, $F(2, 1)$, and $G(1, 0)$. Find the image matrix for a 90° rotation of $DEFG$ about the origin. Graph $DEFG$ and its image.</p> <p>a. Multiply</p> <p>b. Graph:</p> 	<p>10) The vertices of $\triangle ABC$ are $A(2, 4)$, $B(7, 6)$, and $C(5, 2)$. Graph the image of $\triangle ABC$ after a composition of the transformations in the order they are listed.</p> <p>Translation: $(x, y) \rightarrow (x - 4, y - 3)$</p> <p>Reflection: in the x-axis</p> 
<p>11) Determine whether the figure has line symmetry or rotational symmetry. If so, describe the number of lines of symmetry and the rotation(s) that map the figure onto itself.</p> <p>a.</p>  <p>b.</p> 	<p>12) Use the description to draw a figure. If not possible, write <i>not possible</i></p> <p>a. A triangle with exactly three lines of symmetry</p> <p>b. A trapezoid with exactly two <i>lines</i> of symmetry</p>

7) Review Rational Expressions

Factor:

2) $x^3 - 10 + 5x^2 - 2x$	3) $6x^2 + 19x - 7$
4) $2x^2 - 7xy + 3y^2$	5) $36x^2 - 25$
6) $16x^2 - 56x + 49$	7) $64x^3 - 125$
8) $x^2 - 25a^2 + 8x + 16$	9) $3x^3 - 30x^2 + 75x$

Multiply/Divide:

10) $\frac{x^2 - 2x - 8}{x^2 - 9} \div \frac{x - 4}{x + 3}$	11) $\frac{x^2 - 2x + 1}{x^3 + x} \div \frac{x^2 + x - 2}{3x^2 + 3}$
12) $h(x) = \frac{x - 7}{x - 1} \cdot \frac{x^2 - 1}{3x - 21}$	13) $f(x) = \frac{x + 3}{x^2 - 4} \cdot \frac{x^2 - x - 6}{x^2 + 6x + 9}$

Add/Subtract:

14) $\frac{x+2}{2x-3} - \frac{4}{x+3}$	15) $\frac{x+3}{x^2+x-2} + \frac{2}{x^2-1}$
16) $\frac{3}{x+1} + \frac{5}{x-1}$	17) $\frac{x}{x^2-10+25} - \frac{x-4}{2x-10}$

Simplify each complex rational expression:

17) $\frac{1 + \frac{1}{x}}{1 - \frac{1}{x}}$	18) $\frac{\frac{1}{x} - \frac{3}{2}}{\frac{1}{x} + \frac{3}{4}}$
19) $\frac{\frac{1}{x+h} - \frac{1}{x}}{h}$	20) $\frac{\frac{x}{x-2} + 1}{\frac{3}{x^2-4} + 1}$

Simplify each rational expression. Use interval and set-builder notation to represent the domain of each rational function.

1) $f(x) = \frac{x^2-25}{x-5}$	2) $g(x) = \frac{x}{x^2-25}$	3) $f(x) = \frac{x+5}{x^2+25}$
Set-builder notation:	Set-builder notation:	Set-builder notation:
Graph for Interval Notation:	Graph for Interval Notation:	Graph for Interval Notation:
Interval Notation:	Interval Notation:	Interval Notation: